C.U.SHAH UNIVERSITY
Summer Examination-2017

Subject Name: Mathematics - II

Subject Code: 4SC02MTC1   **Branch: B.Sc. (All)**

Semester: 2   **Date: 09/05/2017**   **Time:02:00 To 05:00**   **Marks: 70**

Instructions:
  (1) Use of Programmable calculator & any other electronic instrument is prohibited.
  (2) Instructions written on main answer book are strictly to be obeyed.
  (3) Draw neat diagrams and figures (if necessary) at right places.
  (4) Assume suitable data if needed.

Q-1   **Attempt the following questions:**   **(14)**

a) Find out order and degree of the differential equation   (02)

$$1 + \left(\frac{dy}{dx}\right)^2 = \left(\frac{d^3 y}{dx^3}\right)^2$$

b) Solve: $(D^2 - 1)y = 0$ (02)

c) Evaluate: $\int_0^{\frac{\pi}{2}} \sin^3 x \cos^4 x \, dx$ (02)

d) Find polar form of $1 + \sqrt{3}i$.   (01)

e) Find real and imaginary part of $2e^{-\frac{i\pi}{2}}$ (01)

f) $\lim_{n \to \infty} \sqrt[n]{a} = $ _____ where $a > 0$.   (01)

g) True/ False. If P.I. $= 0$ then the general solution of ordinary differential equation   (01)
is same as the complementary function.

h) Define: Cauchy's sequence.   (01)

i) True/ False. Every convergent sequence is bounded.   (01)

j) Write equation of ellipsoid.   (01)

k) True/ False. If the imaginary part of any complex number is zero then the   (01)
complex number becomes purely real number.

Attempt any four questions from Q-2 to Q-8

Q-2   **Attempt all questions**   **(14)**

a) State and prove De-moivre's theorem   (07)

b) Prove that $(1 + i)^n + (1 - i)^n = 2^{\frac{n}{2}+1} \cos\frac{n\pi}{4}$ (05)

c) Simplify: $\dfrac{(\cos 4\theta - i \sin 4\theta)^4 (\cos 2\theta + i \sin 2\theta)^{-6}}{(\cos 2\theta - i \sin 2\theta)^3 (\cos\theta - i \sin\theta)^{-7}}$.   (02)

Q-3   **Attempt all questions**   **(14)**

a)   Prove that $\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$.   (05)

b)   Find the roots of the equation $z^4 - 1 = 0$.   (05)

c)   Solve: $(D^2 + 2D - 3)y = e^x$.   (04)

Q-4   **Attempt all questions**   **(14)**

a)   Using De-moivre's theorem solve $x^7 + x^4 + i(x^3 + 1) = 0$ (07)

b)   Solve: $x^2 \frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = x^2 \sin(\log x)$.   (07)

Q-5   **Attempt all questions**   **(14)**

a)   Prove that $\cosh^{-1}(z) = \log(z + \sqrt{z^2 - 1})$.   (06)

b)   Find real and imaginary part of i^i.   (05)

c)   Show that $\sin h(ix) = i\sin x$.   (03)

Q-6   **Attempt all questions**   **(14)**

a)   Prove that $I_n = \int \cos^n x\, dx = \frac{\sin x \cos^{n-1} x}{n} + \frac{n-1}{n}I_{n-2}$.   (06)

b)   Evaluate: $\int_0^1 x^6\sqrt{1 - x^2}\, dx$.   (04)

c)   Solve: $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 4y = \cos x$.   (04)

Q-7   **Attempt all questions**   **(14)**

a)   If $f(D)y = e^{ax}$ is given linear differential equation with constant co − efficient then prove that $\frac{1}{f(D)}e^{ax} = \frac{1}{f(a)}e^{ax}$, if $f(a) \neq 0$.   (06)

b)   Show that the equation $2y^2 - 8yz - 4zx - 8xy + 6x - 4y - 2z + 5 = 0$ represents a cone whose vertex is $\left(-\frac{7}{6}, \frac{1}{3}, \frac{5}{6}\right)$.   (06)

c)   Check whether the sequence $\left\{\frac{1}{n}\right\}$ convergent or not.   (02)

Q-8   **Attempt all questions**   **(14)**

a)   Find equation of lines in which the plane $x + 3y - 2$ cuts the cone   (06)
$$x^2 + 9y^2 - 4z^2 = 0.$$

b)   Find equation of cylinder whose generator are parallel to $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and guiding curve $x^2 + y^2 = 16, z = 0$.   (06)

c)   Evaluate: $\frac{1}{D^2}(x^4)$.   (02)